**Simple Co-ordinate geometry problems**

**1.** Find the equation of straight line passing through the point with equal intercepts.

**1.** **Method 1**
 Let the equation of straight line be

 **(a)** If and the straight line passes through ,

 and

 The straight line is .

 **(b)** If , the straight line passes through the origin and

 the equation is .

 **Method 2**

 Let the equation of straight line passing through the point be

 Since x-intercept = y-intercept

 , where

 The equations are

 or .

**2.** Consider a circle with center (4,5) and with radius of 8

 If the tangent lines of the circle has slope . Find :

 **(a)** the points of contact of these tangent lines and the circle,

 **(b)** the equation of the tangent lines.

**2. (a)**

 **Method 1**

 The circle is

 Since the tangent slope is , the slope of the line perpendicular to the tangent is -3.

 Let this diameter line passes through the centre be L. Then

 Solving,

 and

 The required points are

 **Method 2**

 The circle is

 Differentiate with respect to x,

 Since the tangent slope is ,

 We get a straight line

 (This line is **not** the tangent line, but the diameter line passing (4,5) and the point of contact. In fact we just get a line which is good for the given.)

 Substitute in C, we have

 and

 The required points are

 **Method 3**

 The circle in parametric form is

 Since the tangent slope is ,

 ∴

 The required points are .

 **Method 4**

 The circle in parametric form is

 Let the pencil of straight lines with slope be

 Hence, we have:

 Writing in subsidiary angles,

 Since the straight line touches the circle at only one point, we have

 Therefore,

 **(b) Method 1**

Use point-slope form, in **(a)** we get slope = , Points of contact =

 Hence equations of tangent are:

 **If there is no need to find the points of contact, the following methods are also of interest.**

 **Method 2**

 Let the pencil of straight lines with slope be

 It touches the circle

 ,

 Since L touches C at one point,

 Hence the equations of tangent are:

 **Method 3**

 The circle in parametric form is

 Let the pencil of straight lines with slope be

 Hence, we have:

 Writing in subsidiary angles,

 For each c we may have no solution, one solution or two solutions for the straight line.

 Since the straight line touches the circle at only one point, we have .

 (For , the straight line cuts the circle at two points! You get a diameter equation which is parallel to the tangent lines we are interested. Investigate yourselves.))

 Therefore,

 Hence the equations of tangent are:

**3.** Find the value(s) of m such that

 represents the equation of a circle.

**3.**  represents a circle if and only if and the radius > 0.

 Now,

 **(a)** When , , no equation existed and

 is rejected.

 **(b)** When , The circle is

 The radius = which is imaginary and is rejected.

 **(c)** When ,

 The circle is

 The radius is . This satisfies all conditions of the given.

**4.** Find the equation of a circle passing through the intersection points of

 and with the smallest area.

**4.** **Method 1**

 The intersection point of

 are (working steps omitted)

 The smallest circle that can be formed should use these two points as diameter.

 Using the diameter form, the required circle is:

 **Method 2**

 Let the system of circles passing through L and C be

 (**Method 2A)** If this circle has the smallest area, the radius r is also smallest.

 Hence r is smallest when .

 (**Method 2B)** If this circle has the smallest area, the radius r is also smallest.

 Hence the centre of = must be on the line L.

 We therefore have

 Hence r is smallest when .

 The required circle is:

 **Method 3**

 The centre, G, of C =

 Let be the line perpendicular to L and passing through G.

 Gradient of L = and Gradient of .

 Hence,

 Solving , we get . This is the centre of the required circle.

 Let the system of circles passing through L and C be

 The centre is also

 and . The required circle is: