**Simple Co-ordinate geometry problems**

**1.** Find the equation of straight line passing through the point with equal intercepts.

**1.** **Method 1**  
 Let the equation of straight line be

**(a)** If and the straight line passes through ,

and

The straight line is .

**(b)** If , the straight line passes through the origin and

the equation is .

**Method 2**

Let the equation of straight line passing through the point be

Since x-intercept = y-intercept

, where

The equations are

or .

**2.** Consider a circle with center (4,5) and with radius of 8

If the tangent lines of the circle has slope . Find :

**(a)** the points of contact of these tangent lines and the circle,

**(b)** the equation of the tangent lines.

**2. (a)**

**Method 1**

The circle is

Since the tangent slope is , the slope of the line perpendicular to the tangent is -3.

Let this diameter line passes through the centre be L. Then

Solving,

and

The required points are

**Method 2**

The circle is

Differentiate with respect to x,

Since the tangent slope is ,

We get a straight line

(This line is **not** the tangent line, but the diameter line passing (4,5) and the point of contact. In fact we just get a line which is good for the given.)

Substitute in C, we have

and

The required points are

**Method 3**

The circle in parametric form is

Since the tangent slope is ,

∴

The required points are .

**Method 4**

The circle in parametric form is

Let the pencil of straight lines with slope be

Hence, we have:

Writing in subsidiary angles,

Since the straight line touches the circle at only one point, we have

Therefore,

**(b) Method 1**

Use point-slope form, in **(a)** we get slope = , Points of contact =

Hence equations of tangent are:

**If there is no need to find the points of contact, the following methods are also of interest.**

**Method 2**

Let the pencil of straight lines with slope be

It touches the circle

,

Since L touches C at one point,

Hence the equations of tangent are:

**Method 3**

The circle in parametric form is

Let the pencil of straight lines with slope be

Hence, we have:

Writing in subsidiary angles,

For each c we may have no solution, one solution or two solutions for the straight line.

Since the straight line touches the circle at only one point, we have .

(For , the straight line cuts the circle at two points! You get a diameter equation which is parallel to the tangent lines we are interested. Investigate yourselves.))

Therefore,

Hence the equations of tangent are:

**3.** Find the value(s) of m such that

represents the equation of a circle.

**3.**  represents a circle if and only if and the radius > 0.

Now,

**(a)** When , , no equation existed and

is rejected.

**(b)** When , The circle is

The radius = which is imaginary and is rejected.

**(c)** When ,

The circle is

The radius is . This satisfies all conditions of the given.

**4.** Find the equation of a circle passing through the intersection points of

and with the smallest area.

**4.** **Method 1**

The intersection point of

are (working steps omitted)

The smallest circle that can be formed should use these two points as diameter.

Using the diameter form, the required circle is:

**Method 2**

Let the system of circles passing through L and C be

(**Method 2A)** If this circle has the smallest area, the radius r is also smallest.

Hence r is smallest when .

(**Method 2B)** If this circle has the smallest area, the radius r is also smallest.

Hence the centre of = must be on the line L.

We therefore have

Hence r is smallest when .

The required circle is:

**Method 3**

The centre, G, of C =

Let be the line perpendicular to L and passing through G.

Gradient of L = and Gradient of .

Hence,

Solving , we get . This is the centre of the required circle.

Let the system of circles passing through L and C be

The centre is also

and . The required circle is: